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Duncan M. Y. Sommerville

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DUNCAN M. Y. SOMMERVILLE.

By GEORGE BRUCE HALSTED.

It has been said the heroic age of non-euclidean geometry is passed, since long gone are the days when Lobachevski flinched into calling his system "imaginary geometry," and, I might add, Gauss kept a more cowardly silence because, as he confesses, he "fears the outcry of the Boeotians," (proverbial for stupidity).

But there are other kinds of heroism beside that typified by the biting blade of Bolyai's sabre, and here I set forth still another paladin, Duncan Sommerville, whose triumph in his *Bibliography of Non-Euclidean Geometry* I can, perhaps, better than any one else appreciate, since I myself wrestled for years at this very labor of Hercules.

Lobachevski was of Kazan, city of the Golden Horde of Tatars on Mother Volga. John Bolyai was born at Kolozsvâr, birthplace of Matthias Corvinus, chosen king of Hungary by forty thousand Magyars assembled upon the ice of the frozen Danube, and the young Jânos grew up on the banks of Maros the mordicant. Duncan Sommerville was born in India, in Rajputana, land named from the warrior caste, analogue of the Samurai and like them unorthodox.

In my Biography of Hoüel, of a very old protestant family of Normandy, I say: "The key to his whole mental life was this old protestant blood, which means so much in a Roman Catholic country." The women of his family, who had probably never heard of Sherlock Holmes, wrote me their amazement that I could so have penetrated their family secrets, and used the significant phrase: In science there are no frontiers!

Duncan Sommerville, born November 24th, 1879, was eventually sent to Perth Academy, Scotland, and in 1896 entered St. Andrews University, third Bursar, where he won medals in mathematics, Ordinary and Honors, Greek, botany, and held the Ramsay Scholarship and the Berry Scholarship in mathematics and natural philosophy. In 1900 he graduated with the degree B. Sc. and M. A. with First Class Honors in mathematics and natural philosophy.

He then pursued the first two years of a medical course, but in 1902 began teaching applied mathematics (dynamics). The following year he was appointed assistant to the professor of mathematics at St. Andrews University and lecturer on applied mathematics.

In 1905 he won the degree of D. Sc. for the thesis: "Networks of the Plane in Absolute Geometry," thus using in his thesis title a phrase of my making; for the designation "Absolute Geometry" was first used by myself as a rendering for John Bolyai's phrase *scientiam spatii absolute veram,* in which I have always gloried as showing the magnificent nerve of the young Magyar hero, victim of the meanness of Gauss, as was also his own son who passed his life an exile here in Colorado.

Dr. Sommerville is at present lecturer in mathematics and in applied mathematics at the University of St. Andrews. He is a Fellow of the Royal Society of Edinburgh, and president of the Edinburgh Mathematical Society. He is a member of the Circolo Matematico di Palermo, and of the Perthshire Society of Natural Science.

At the Sheffield meeting of the British Association (1910) at which he read a paper on Non-Euclidean Bibliography, a committee was appointed to consider the advisibility of drawing up a report on the subject of non-euclidean geometry. Dr. Sommerville was made secretary, and Dr. H. F. Baker, of Cambridge, chairman. Other members were: Professor Chrystal of Edinburgh, Dr. A. N. Whitehead of Cambridge (in whose article, Geometry, in the *Encyclopaedia Britannica,* my name occurs thrice), Professor Burnside of Greenwich, and Sir Robert S. Ball of Cambridge. That this Report would be greatly appreciated may be argued from the fact that my own report on non-euclidean geometry to the American Association in 1899, and my supplementary report in 1901, were each published in full four times over, thus vieing in fame with my Bibliography of Hyperspace and Non-Euclidean Geometry, of which the University of St. Andrews advertising in *Science* (No. 865, p. vii) its publication of Sommerville's *Bibliography of Non-Euclidean Geometry* says: "This work is regarded as a continuation of Professor Halsted's often quoted bibliography which was published in the *American Journal of Mathematics.*"

In the session of 1908-9 a mathematical and physical society was instituted in the University of St. Andrews, of which Dr. Sommerville is still president. It exists for the furtherance of an interest in mathematical and physical subjects, and has been very successful, interesting papers being read at fortnightly intervals by members of the staff and by students.

Dr. Sommerville writes me: "I find myself going into the elements of non-euclidean geometry in Honors lectures on geometry, for I think that one cannot arrive at the true nature of ordinary geometry without a comparison with the non-euclidean systems."

How very like is this to the quotation from Professor Eduard Study of Bonn, which I placed at the head of my paper "The Value of Non-

Euclidean Geometry'' in *The Popular Science Monthly* for November, 1905: ''Among conditions to a more profound understanding of even very elementary parts of the euclidean geometry, the knowledge of the non-euclidean geometry can not be dispensed with.''

And now finally of the tremendous achievement to which all this is but a prelude, Sommerville's Bibliography of Non-Euclidean Geometry, suffice it to say that it is an immortal monument, showing to all what we of the cult have ever realized, that here we face one of the very few germinal ideas whose creation has made our modern world.

John Bolyai published but two dozen pages; what I have long called the most extraordinary two dozen pages in the whole history of thought. Here we see shown us a list of their fruit more than four thousand articles and books dealing with this subject.

The following is a bibliography of Sommerville's productions:

1905. ''Semi-regular networks of the plane in absolute geometry.'' Edinburgh, Trans. R. Soc., 41, 725-747 (12 plates); Edinburgh, Proc. R. Soc., 25, 392-394 [abstract].

''On the number of independent conditions involved in the vanishing of a rectangular array.'' Edinburgh, Proc. Math. Soc., 24, 2-6.

1906. ''On the distribution of the proper fractions.'' Edinburgh, Proc. R. Soc., 26, 116-129.

''On the classification of frequency-ratios.'' Biometrika, Cambridge, 5, 179-181.

''Questions 15894, 15903, 15906, 15948, 15925.'' Math. Questions, *Educational Times*, London, 10, 34, 40, 71, 83-84, 93-94.

1907. ''On links and knots in euclidean space of *n* dimensions.'' *Messenger Math.*, Cambridge, 36, 139-144.

''On certain projective configurations in space of *n* dimensions and a related problem in arrangements.'' Edinburgh, Proc. Math. Soc., 25, 80-90.

''Questions 15986, 15999, 16009.'' Math. Questions, *Educational Times*, London, 11, 24-25, 55-56, 57-61. [Q. 16009, proposed by H. Bateman, ''On systems of hyperspheres, each touching two others in succession.'' Generalized for euclidean or non-euclidean space of any dimensions.]

1908. ''Sunset and twilight curves, and related phenomena'' Edinburgh, Proc. R. Soc., 28, 311-337 (3 plates).

''Sunset and twilight.'' Perth, Proc. Soc. Nat. Sci., 4, clxxxiv-clxxxvi (1 plate).

''Question 15951.'' Math. Questions, *Educational Times*, London, 12, 24-25.

1909. ''On certain periodic properties of cyclic compositions of nnmbers.'' London, Proc. Math. Soc., (2) 7, 263-313.

1910. "A problem in voting." Edinburgh, Proc. Math. Soc., 28.

"Classification of geometries with projective metric." Edinburgh, Proc. Math. Soc., 28, 25-41.

"Elementary considerations relating to the absolute." Edinburgh, Proc. Math. Soc., 28, 65-72.

"Note on the geometries in which straight lines are represented by circles." Edinburgh, Proc. Math. Soc., 28, 81-94.

"The early history of non-euclidean geometry." *Nature*, London, 84, 172.

"On the need of a non-euclidean bibliography." Rep. Brit. Assoc., 80 (Sheffield), 531-532.

1911. "The numerically greatest term of a binomial expansion." Edinburgh, Math. Notes, No. 7, 74-77.

"Concrete representations of non-euclidean geometry." In a Collection of Scientific Papers published by the University of St. Andrews to commemorate the 500th anniversary of its founda-

"Bibliography of non-euclidean geometry, including the theory of parallels, the foundations of geometry, and space of n dimensions." London: Harrison, pp. 400.

NOTES ON GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE OF INTEGERS.

By **BENJAMIN FRANKLIN YANNEY**, Wooster University.

I. PROPERTIES OF QUOTIENTS.

Let $q_1, q_2, \ldots, q_n$ be the respective quotients obtained by dividing the positive integers $a_1, a_2, \ldots, a_n$ by their greatest common divisor D; and $q'_1, q'_2, \ldots, q'^n$ the respective quotients obtained by dividing L, the lowest common multiple of the integers $a_1, a_2, \ldots, a_n$, by the integers, in turn. Hence

$$q_1 = a_1 \div D;\ q_2 = a_2 \div D;\ \ldots;\ q_n = a_n \div D \ldots (1);$$

$$q'_1 = L \div a_1;\ q'_2 = L \div a_2;\ \ldots;\ q'_n = L \div a_n \ldots (2).$$

The two sets of quotients possess the following properties:

1. Neither set of quotients has a factor common to all the numbers in the set. For, otherwise, D would not be the greatest common divisor nor L the least common multiple of the numbers $a_1, a_2, \ldots, a_n$.

2. $q_1 q'_1 = q_2 q'_2 = \ldots = q_n q'_n = L \div D$.

This result is obtained by multiplying, member by member, the cor-